

Tangent Line Exercise

Find all $a, b, c \in R$ for which the functions $f(x) = x^2 + ax + b$ and $g(x) = cx - x^2$ satisfy $f(1) = g(1) = 3$ and have the same tangent line at the point $(1, 3)$.

Solution

I calculate the slopes of the tangent lines at the point $(1, 3)$:

$$f' = 2x + a = 2 + a$$

$$g' = c - 2x = c - 2$$

We then know that $2 + a = c - 2$, so $4 + a = c$. Now we evaluate the functions at the corresponding points:

$$f(1) = 1 + a + b = 3$$

$$g(1) = c - 1 = 3$$

With this, I find that $c = 4$. Therefore $a = 0$. And finally $b = 2$. The functions are:

$$f(x) = x^2 + 2$$

$$g(x) = 4x - x^2$$

The tangent line of $f(x)$ is:

$$y = 2x + o$$

To find the y-intercept, I substitute in the point:

$$3 = 2 + o$$

Therefore $o = 1$:

$$y = 2x + 1$$

I find the tangent line of g :

$$y = 2x + o$$

I find the y-intercept again:

$$3 = 2 + o$$

Therefore $o = 1$, and the tangent lines of both functions are the same.