

## Tangent Line Exercise

Find all  $a, b, c \in \mathbb{R}$  for which the functions  $f(x) = x^2 + ax + b$  and  $g(x) = cx - x^2$  satisfy  $f(1) = g(1) = 3$  and have the same tangent line at the point  $(1, 3)$ .

## Solution

I calculate the slopes of the tangent lines at the point  $(1, 3)$ :

$$f' = 2x + a = 2 + a$$

$$g' = c - 2x = c - 2$$

We then know that  $2 + a = c - 2$ , so  $4 + a = c$ . Now we evaluate the functions at the corresponding points:

$$f(1) = 1 + a + b = 3$$

$$g(1) = c - 1 = 3$$

With this, I find that  $c = 4$ . Therefore  $a = 0$ . And finally  $b = 2$ . The functions are:

$$f(x) = x^2 + 2$$

$$g(x) = 4x - x^2$$

The tangent line of  $f(x)$  is:

$$y = 2x + o$$

To find the y-intercept, I substitute in the point:

$$3 = 2 + o$$

Therefore  $o = 1$ :

$$y = 2x + 1$$

I find the tangent line of  $g$ :

$$y = 2x + o$$

I find the y-intercept again:

$$3 = 2 + o$$

Therefore  $o = 1$ , and the tangent lines of both functions are the same.